A Traveler's Tale

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Visualize, Vocabulary Organizer, Create Representations

My Notes

Jonathan Swift wrote the classic tale *Gulliver's Travels*. The story describes the adventures of Lemuel Gulliver, a ship's doctor who becomes stranded in many strange places. In one of those places, Lilliput, Gulliver finds that he is a giant compared to the people and the world around him. During another voyage, Gulliver is abandoned in another land, Brobdingnag, where he is as small to the inhabitants as the Lilliputians were to him.

The story never says how tall Gulliver is, but it does tell how the heights of the Lilliputian people and the heights of the people from Brobdingnag compare to Gulliver's height. The many descriptions of size in this tale provide ways to explore the magnitude of numbers. For this activity, assume that Gulliver is 6 feet tall.

1. What do you think the word *magnitude* means?

2. Since Gulliver is so much bigger than the Lilliputians, he consumes more food than 10^3 Lilliputians do. How many Lilliputians does this number represent?

3. If a person from Brobdingnag is 10 times taller than Gulliver, how tall is this person?

4. Write an expression using Gulliver's height and a power of 10 to represent the height of a person in Question 3.

5. If a person is 10 times taller than a person from Brobdingnag, how many times taller than Gulliver is this person?

6. Write an expression using Gulliver's height and a power of 10 to represent the height of the person in Question 5.

In Questions 4 and 6, each expression is the product of a factor and a power of 10 with an exponent that is a positive integer.

My Notes

SUGGESTED LEARNING STRATEGIES: Think Aloud, Think/Pair/Share, Identify a Subtask, Discussion Group, Look For a Pattern, Quickwrite, Marking the Text, Question the Text, Vocabulary Organizer

7. Find the value of each expression. Each expression is the product of a factor and a power of 10 with an exponent that is a positive integer.

a. 6×10^4

b. 15×10^3

c. 2×10^6

d. 3.2×10^5

e. 43.2×10^3

8. Describe any patterns you notice in evaluating expressions like those in Question 7.

9. After his boat capsizes in a violent storm, Gulliver swims ashore to Lilliput and falls asleep. When he wakes, Gulliver finds he has been tied to the ground and can only look up into the bright sun. The sun has a diameter of 1.39×10^9 m and a mass of 2.0×10^{30} kg. Use the patterns you described in Question 8 to rewrite the diameter and mass of the sun.

CONNECT TO LANGUAGE ARTS

A boat that *capsizes* has turned over or tipped over.

READING MATH

The expression $a \cdot 10^n$ is read "a times 10 raised to the nth power."

The measurements, 1.39×10^9 m and 2.0×10^{30} kg, are written in **scientific notation**, and the answers to Question 9 are written in **standard form**. A number is written in scientific notation when it is expressed in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

10. Explain why someone would want to write these numbers in scientific notation instead of standard form.

ACADEMIC VOCABULARY

scientific notation

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Look for a Pattern, Identify a Subtask, Work Backward

My Notes

11. Tell whether each expression is written in scientific notation. If it is not written in scientific notation, explain why not and rewrite in scientific notation if possible.

a. 6×10^4

b. 15×10^3

c. 2×10^6

d. 3.2×10^5

e. 43.2×10^3

f. $5.9 \times 10^{0.5}$

12. Convert each number from standard form to scientific notation. Then work backward from your answer to check your work.

a. 25,000,000,000 **b.** 6000

c. 43,600,000 **d.** 16,000

13. Large numbers can also be named with words. For example, 9,000,000,000 in scientific notation is 9×10^9 and has the name 9 billion. This table shows names for some very large numbers.

Standard Form	Power of 10	Name
1,000	10^3	Thousand
1,000,000	10^6	Million
1,000,000,000	10^9	Billion
1,000,000,000,000	10^{12}	Trillion
1,000,000,000,000,000	10^{15}	Quadrillion
1,000,000,000,000,000,000	10^{18}	Quintillion
1,000,000,000,000,000,000,000	10^{21}	Sextillion
1,000,000,000,000,000,000,000,000	10^{24}	Septillion
1,000,000,000,000,000,000,000,000,000	10^{27}	Octillion
1,000,000,000,000,000,000,000,000,000,000	10^{30}	Nonillion
1,000,000,000,000,000,000,000,000,000,000,000	10^{33}	Decillion
There is not enough space to write this number.	10^{100}	Googol

My Notes

SUGGESTED LEARNING STRATEGIES: Think Aloud, Create Representations, Look for a Pattern, Work Backward, Identify a Subtask, Quickwrite

Use what you have learned to complete this table.

Standard Form	Power of 10	Name
7,400,000,000,000,000	7.4×10^{15}	7.4 quadrillion
	3×10^{3}	
1,200,000,000		
		5 trillion
9,000,000		

14. The kingdom of Lilliput is said to have an area of 24 million square miles. Write this amount using scientific notation.

15. Convert each number from scientific notation to standard form. Check your work.

a. 5.2×10^{4}

b. 4.23×10^{6}

c. 2×10^{3}

d. 1.03×10^{4}

16. The fictional land of Brobdingnag had an area of 1.8×10^{7} square miles. In its army were 32,000 cavalry and 2.07×10^{5} soldiers. Order these numbers from greatest to least.

17. Explore the products of powers of 10.

a. Write each product in expanded form. Then express the product as a power of 10. The first one has been done.

$10^{2} \cdot 10^{4} = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10) = 10^{6}$

$10^{3} \cdot 10^{2} =$

$10^{5} \cdot 10^{3} =$

$10^{4} \cdot 10 =$

b. Describe a pattern that relates the exponents of the factors and the products for each entry in the list in Part a.

SUGGESTED LEARNING STRATEGIES: Think Aloud, Quickwrite, Create Representations, Group Presentation, Identify a Subtask

My Notes

c. Based on the pattern you described in Part b, write the missing exponent in this equation to show a rule for finding a product with exponents.

$$10^m \times 10^n = 10^{\_\_\_\_}$$

18. Suppose that a person from Brobdingnag consumes an average of 18,000 calories a day and that the land of Brobdingnag has a civilian population of 30,000.

a. Express the number of calories consumed by one person in one day in scientific notation.

b. Express the civilian population of Brobdingnag in scientific notation.

c. Find the total number of calories consumed in one day in the land of Brobdingnag. Express the total number of calories in standard form and in scientific notation.

d. Explain how to use the expressions from Parts a and b to find the answer for Part c.

19. Simplify each expression. Write the answer in scientific notation.

a. $(9 \times 10^5)(3 \times 10^4)$

b. $(1.6 \cdot 10^8)(3 \cdot 10^4)$

c. $(4 \cdot 10^8)(6 \cdot 10^5)$

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share
Look for a Pattern, Quickwrite, Create Representations

My Notes

20. Now you can compare the products of each expression from Question 19 with those found using a calculator.

a. Write each output as shown on your calculator.

a. $(9 \times 10^5)(3 \times 10^4)$

b. $(1.6 \times 10^8)(3 \times 10^4)$

c. $(4 \times 10^8)(6 \times 10^5)$

b. Explain what the outputs on your calculator mean.

21. Find each product. Express the answers in decimal form.

a. $2 \cdot \frac{1}{10}$

b. $15 \cdot \frac{1}{1000}$

c. $6 \cdot \frac{1}{10{,}000}$

d. $3.2 \cdot \frac{1}{100}$

e. $43.2 \cdot \frac{1}{1000}$

22. Describe any patterns you notice in evaluating expressions like those in Question 21.

23. Explore the quotients of powers of 10.

a. Write each quotient in this list in expanded form, simplify, and then express the quotient as a single power of 10. The first entry of the list has been completed for you.

$$\frac{10^5}{10^2} = \frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = 10 \cdot 10 \cdot 10 = 10^3$$

$$\frac{10^8}{10^6} =$$

$$\frac{10^7}{10^2} =$$

$$\frac{10^5}{10} =$$

SUGGESTED LEARNING STRATEGIES: Think Aloud, Discussion Group, Create Representations, Quickwrite, Look for a Pattern

My Notes

b. Describe a pattern relating the exponents in each row of the list on page 68.

c. Based on the pattern you described in Part b, write the missing exponent in this equation to show a rule for quotients with exponents.

$$\frac{10^m}{10^n} = 10$$

d. Complete this list as you did in Part a, but use the rule from Part c to express each quotient as a power of 10. The first entry in the list has been completed for you.

$$\frac{10^4}{10^6} = \frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = \frac{10}{10 \cdot 10 \cdot 10} = \frac{10^1}{10^3} = 10^{-2}$$

$$\frac{10^2}{10^5} =$$

$$\frac{10^3}{10^7} =$$

e. Based on your work in Part d, what fractional value do you think mathematicians give to 10^{-1}, 10^{-4}, and 10^{-n}?

f. Complete this list as you did in Part d. Remember to use the rule from Part c to express the quotient as a power of 10.

$$\frac{10^5}{10^5} =$$

$$\frac{10^3}{10^3} =$$

g. Based on your work in Part f, what meaning do you think mathematicians give to 10^0?

24. If the Lilliputians are 10 times shorter than Gulliver, how tall is a Lilliputian?

My Notes

SUGGESTED LEARNING STRATEGIES: Create Representations, Identify a Subtask, Work Backward, Think/Pair/Share, Marking the Text

25. In terms of Gulliver's height, express the height of a Lilliputian, using scientific notation.

26. In terms of Gulliver's height, use scientific notation to express the height of an insect if the insect is 10 times shorter than a person from Lilliput.

27. Convert each number from standard form to scientific notation. Check your work.

a. 0.125

b. 0.00006

c. 7

d. 0.000000000025

28. Explain why someone would want to write these numbers in scientific notation instead of standard form.

29. Convert each number from scientific notation to standard form. Check your work.

a. 5.2×10^{-4}

b. 4.23×10^{-6}

c. 2.5×10^{0}

30. Order these numbers from least to greatest.

$0.0051, 8 \times 10^{-4}, 1.89 \times 10^{-3}, 0.00079$

31. Suppose that all of the 500,000 Lilliputians consume a total of 15,000 gallons of water in one day.

a. Express the total number of gallons of water consumed by all of the Lilliputians in one day in scientific notation.

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Think/Pair/Share, Identify a Subtask, Discussion Group, Look for a Pattern

My Notes

b. Express the population of Lilliput in scientific notation.

c. Find the average number of gallons of water consumed by one Lilliputian in one day. Express answer in both standard form and scientific notation.

d. Show how to use the expressions from Parts a and b to find the answer for Part c.

32. Simplify each expression. Write answers in scientific notation.

a. $\frac{6 \times 10^5}{3 \times 10^4}$

b. $(2 \times 10^{-5})(3 \times 10^4)$

c. $(4.2 \times 10^8)(3 \times 10^{-5})$

d. $\frac{3.2 \times 10^5}{4 \times 10^8}$

33. Think about the value of each expression.

4.3×10^3	3.8×10^{-5}	2.4×10^{12}	3.0×10^0
2.2×10^{-2}	7.8×10^{-4}	7.1×10^0	9.8×10^5
6.4×10^{-3}	3.8×10^{-14}	6.4×10^8	4.8×10^0

a. Place each expression in the appropriate column.

Between 0 and 1	From 1 to 10	10 and greater

SUGGESTED LEARNING STRATEGIES: Quickwrite

My Notes

b. Explain what you notice about the exponents in the scientific notation form of the numbers you sorted.

Numbers between 0 and 1:

Numbers from 1 to 10:

Numbers 10 and greater:

34. On a separate sheet of paper, explain how this activity about *Gulliver's Travels* involves magnitude of numbers and scientific notation.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Is 10.2×10^4 written in scientific notation? Explain.

2. Copy and complete.

Standard Form	Scientific Notation	Name
2,300,000,000		
	3.4×10^3	
		9 million

3. The following table shows the attendance for a year at four major league baseball stadiums.

Yankees	Mariners	Red Sox	Dodgers
3.8×10^6	2,000,000	2.5 million	3.2×10^6

Order the attendances from greatest to least.

4. Simplify each expression. Write the answers in scientific notation.

a. $(2.2 \times 10^5)(4 \times 10^7)$

b. 35,000 • 9,000,000,000

c. $(8.1 \times 10^{12})(5.3 \times 10)$

5. Copy and complete.

Standard Form	Scientific Notation
0.00009	
	1.7×10^{-3}
	6.99×10^{-7}
0.00086	

6. Wire 1 has a diameter of 9×10^{-2} inches. Wire 2's diameter is 2.4×10^{-3} inches, and Wire 3 is 0.0023 inches in diameter. Order the wire diameters from smallest to largest.

7. Simplify each expression. Write the answers in scientific notation.

a. $(6.5 \times 10^{-13})(2 \times 10^{-4})$

b. $\frac{2.7 \times 10^4}{1.2 \times 10^9}$ **c.** $\frac{4.2 \times 10^{10}}{2.1 \times 10^3}$

8. MATHEMATICAL PRACTICES You used scientific notation to discuss size in Jonathan Swift's *Gulliver's Travels*. Describe how scientific notation aids the discussion of very small or very large numbers. Provide an example.